Part 1: "Surfer Speed vs. Wave Speed and Peel Angle"

by Larry Goddard
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When discussing motion and energy, most readers would benefit from having a basic understanding of the Physics of Motion and Energy. For those who haven't already acquired that knowledge and understanding, it is recommended that they go to any decent library and study a few textbooks on the Physics of Motion, written for students at least at the High School level. A little brush up on 1st-year Algebra is in order for people long out of school. You want to know how to manipulate an equation so you can 'solve for an unknown.'

But first, a few basic concepts need to be covered, regarding Energy:

Energy is defined as the 'Ability to do Work', that is to move something, or change something. Energy can be in the form of Electromagnetic Radiation (such as heat, light, radio waves, or X-rays). Sound waves and water waves also carry or transport energy. Energy can also be Electrical, Chemical, or Nuclear.

'Work' is performed when a Force moves something through a Distance. If you put your shoulder to the wall of a building and try to move it, you may be able to work up a sweat, but if the building doesn't move, you haven't accomplished any 'work'. Try lifting a 1000-lb barbell: if it doesn't move, no work has occurred. But lift a 100-lb barbell 2 feet off the floor, and you have just produced 200 ft-lbs of 'work'.

If you weight 165 lbs and climb up a 10-foot flight of stairs, you have performed 1650 ft-lbs of work.

If you can run up a 10-ft ramp or climb that 10-ft flight of stairs in 3 seconds, you have just performed 1650 foot-pounds of work in 3 seconds. That works out to 550 ft-lbs per second. The 'Rate of Work' is defined as "POWER". The unit of power in this example is 'ft-lbs per second' (English units of measure).

If you use Metric Units, if you use Kilograms for Force, and Meters for distance, the unit of Work is Kilogram-Meters, and that is equivalent to the amount of Energy measured in Joules. Joules = KG-m.

When it was determined in Merry Olde England that a draft horse could lift large buckets of water from a flooded coastal mine, using rope and pulleys, performing that work at a rate of 550 ft-pounds per second, THAT became the standard unit of Power used for rating steam engines: One Horsepower = 550 ft-lbs per second, or 33,000 ft-lbs per minute

So, the 165-lb person climbing a 10-ft flight of stairs in only 3 seconds has just produced 1 HP for a short period of time. That kind of power output is unsustainable for human beings. The closest any athlete is ever likely to get to working at the rate of 1 HP is for a world-class bicycle racer in a sprint. But, he better have a BIG pair of lungs! Think Tour d' France: Does the name "Lance" sound familiar?

Energy is equivalent to Work, and is measured using the same units as Work. The units can be expressed in Metric units, (Joules), or the older "English" measure units that I have been using above (that is, ft-lbs). The units are of Force times Distance. Work = F x D = ft-lbs.

But, Energy can be 'Potential Energy', or 'Kinetic Energy'. For a surfer on a wave, 'Potential Energy' is the 'Energy of Position', that is, the energy acquired when being lifted up by a wave to a position higher than sea level, (where he started out), maybe all the way up to the TOP of the wave. Here, Potential Energy = Weight times your Height above sea level (asl) up on the wave face. So, PE = W x H.

'Kinetic Energy' is a property of a moving body...the Energy of Motion. When you drop in on a wave, you trade your energy of position for moving energy...i.e., for speed in the bottom turn. Then you use some of that moving energy to climb back up higher on the wave, ready for the next maneuver.

The same principal applies to a roller coaster car full of riders: you trade Potential Energy for speed, or Kinetic Energy, which you then use to climb up to the top of the next rise.

Kinetic Energy is proportional to the Mass of a body and to the SQUARE of the Velocity. The formula for KE is given as: KE = (1/2) times Mass x Velocity Squared, or, KE = (1/2)MV^2

Note that any physical body has Mass, but has NO 'Weight' in the absence of a gravitational field acting or 'pulling' on it. It still has 'Inertia', though, which is a property of Mass, and is defined as the resistance to any change in its state of motion, that is, a resistance or 'reluctance' to speed up or slow down, or to change its direction of motion.

If a Force is applied to a body with mass, the resulting change in its motion, or Acceleration (whether positive or negative, depending on the direction of the applied force, relative to its direction of motion), is proportional to to the Force applied, and Inversely Proportional to its Mass. So, as Isaac Newton discovered, the 'rate of change' in velocity, or acceleration of a body, "a" = F/M, so: F = Ma

Experimenters like Galileo and Newton had already established that, if a body started at rest, then the Velocity achieved by a body accelerating at a uniform rate was directly proportional to the Acceleration and the Time elapsed since starting. That is, v = at.

But, we also know that the acceleration, a = F/M, so we can substitute "F/M" for the "a" in the "v = at" formula, and we get the following result: v = (F/M)t, or, v = Ft/M, which can be rewritten as:

Mv = Ft

The "Mv" side of the equation is known as Momentum, and the "Ft" side of the equation is known as "Impulse". So, we can see that, Impulse = Momentum.

"Impulse" is a measure of how long a Time you need to apply a Force to a body to give it a given velocity; and "Momentum" is a measure of how long a Time it would take for a given retarding Force to bring that body back to a state of rest.

Note that Kinetic Energy can also be a measure of how FAR a moving body would travel while a given Retarding Force is acting on it: If KE = F times Distance, then Distance = KE/Force.

If you're launching a rocket (or a dragster), Force is the rocket motor's Thrust, and Time is the Burn Time (or ET, elapsed time at the dragstrip).

Now, once you've gone through the timing lights at the end of the strip at 300 MPH, you need to deploy the drag 'chute to bring the dragster back to a safe stop. That Retarding Force has to be able to bleed off the speed that was attained at the lights. The Time required to do that depends on the Momentum of the dragster going thru' the lights. The Distance it takes to stop the dragster depends on its Kinetic Energy when it went thru' the lights, and the Retarding Force produced by the 'chute and brakes.

Kinetic Energy is a measure of the amount of Work that is required to accelerate a body of mass M, by a given Force F, to a given velocity "v". It's just the product of the Force times the Distance the force acted.

So, if a body starts out at rest (initial velocity = zero), and if the acceleration "a" is at a uniform rate, then the 'Average Velocity' is just half the final velocity, i.e., $V_{avg} = v/2$. The distance travelled, or Space "s" covered by that accelerating body in Time "t" is the Average Velocity times the time interval, "t". So, we have: s = Vavg times t, or $s = (1/2)vt$

But, we know that $v = at$, so, by substituting "at" in place of "v", we get: $s = (1/2)(at)t =(1/2)at^2$ Thus, the accelerating body will cover a Space of: $s = (1/2)at^2$

Now, since Work = Force times Distance, or F times s, we can see that Work = Fs, = $F(1/2)at^2$ But, we know that $F = Ma$, so Work = $(Ma)(1/2)at^2$, or Work = $(1/2)M(a^2)(t^2) = (1/2)M(at)^2$.

But, again, remembering that $v = at$, then, $v^2 = (at)^2$, so, we have: Work = $(1/2)Mv^2$

This is identical to the formula for Kinetic Energy! $KE = (1/2)Mv^2$ So, KE = Work! This is the proof that Kinetic Energy is the same as the Work performed on the accelerating body.

In a gravitational field, where $a = g$, the attractive Force of the gravitational field, we see that $F = Mg$. We experience that Force as 'Weight', so we can rewrite the equation $F = Mg$ as: $W = Mg$

Now, solving for M, we get the equivalent expression for determining Mass in any gravitational field with a known acceleration of gravity = 'g': so, $M = W/g$

If a wave lifts your body and surfboard with 'combined weight' W a distance equal to the Height of the wave above sea level, the 'Potential Energy' gained = (Total Weight) x (Wave Height), so...

PE, ft-lbs = Work = W,lbs times H,ft or, PE = WxH or, simply: PE = WH

Now, let's find the Energy of Motion: 'Kinetic Energy'

We already know $KE = (1/2)MV^2$, so $KE = (1/2)(W/g)V^2$

For our purposes, here, there are two kinds of 'Reference Systems' involved here:
1) The motion of the surfer on the wave face, with the Wave Form as the stationary reference system, and...
2) The motion of the surfer on a moving Wave Form, both moving together over the stationary bottom.

I will use the second reference system, calculating motion relative to the 'Bottom' in the surf zone (sand bar, reef, whatever, below the breaking waves).

Although we are dealing with Velocities, which are Vector quantities of motion, (describing both the Speed AND Direction), I will be referring to 'Speeds' only, which are simple 'Scalar' quantities, so I see no real need to resort to Vectors here, which is a legitimate and proper method, but unnecessarily confusing for most people who are not mathematicians.

Instead, I'll use the simplest, most basic Trigonometric functions as they relate to the sides and interior angles of a Right Triangle.

My reasoning is that the Wave Speed can be represented by the short side of a right triangle, and the Curl Speed by the other, longer side of the triangle, moving at right angles to the Wave Speed. Then, the Hypotenuse of the triangle represents the Resultant Speed between the two motions which are acting at right angles to each other. So, if the Surfer on the wave stays in the same relative position to the Curl, the Hypotenuse of the triangle represents HIS speed, which I'll call "Surfer Speed", or Vs.

If a surfer were proned out at the bottom of a wave, or just going 'straight off', not angling at all, he would only be moving at the Wave Propagation Speed in the surf zone, and the Ride Angle (measured AWAY from going 'straight-off', i.e., WITH the wave in the same direction the wave is moving) would be Zero Degrees. The faster the wave curl is peeling across the wave, the larger the "Peel Angle". That Peel Angle depends on the Swell Direction, relative to a given reef or bottom contour, and on the shape of the reef or bottom in the surf zone ("Bathymetry").

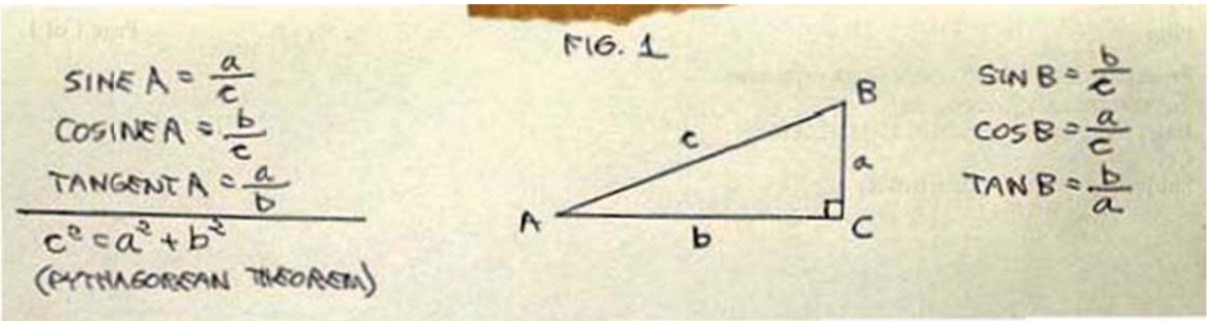
How 'hollow' the curl is depends on the Bottom Slope in the surf zone. Thus, Rapid rise = hollow break! Most good surf spots have a bottom slope of around 1 in 30. Easy, beginner surf spots have a flatter slope, maybe 1 in 100.

There's a personal reason I choose to use 'straight-off' for Zero Degrees: I like to draw graphs with the Dependent Variable INCREASING (rising) as the Independent Variable INCREASES to the right. So, as the curl or Peel Angle INCREASES, the Curl Speed ALSO increases, and the surfer has to go FASTER just to make the wave. Or, the faster the wave peels across, the faster the surfer CAN go, if he has a fast enough board. So, on my graphs of "Surfer Speed vs. Wave Speed" (or Peel Angle), as X increases, Y increases. So, if Surfer Speed is the Dependent Variable, Y. It 'depends' on the Independent Variable, X, (which can be either Wave Speed or Peel Angle).

SIMPLE!...so please, don't anybody argue with me, OK? It's just a personal preference. The results are the same regardless of which way you choose to measure Peel Angle or Ride Angle. You just use the Sine or the Cosine depending on which way you want to do it. (Explanation follows):

For a Right Triangle (with the two sides at 90 degrees to each other), the SUM of the other two angles must be also 90 degrees. All 3 interior angles of ANY triangle can only add up to a total of 180 degrees. So, if you know ONE of the interior angles of a right triangle, the other is EXACTLY (90 - the known angle).

Figure 1. Standard Labelling of a Right Triangle



If we choose to label the ANGLE formed between the hypotenuse and the SHORT side of a right triangle "Angle B", then we would label the SIDE OPPOSITE angle B, as "Side b".
Then, if we label the other angle formed between the hypotenuse and the LONG side "Angle A", then the SIDE OPPOSITE Angle A becomes "Side a".

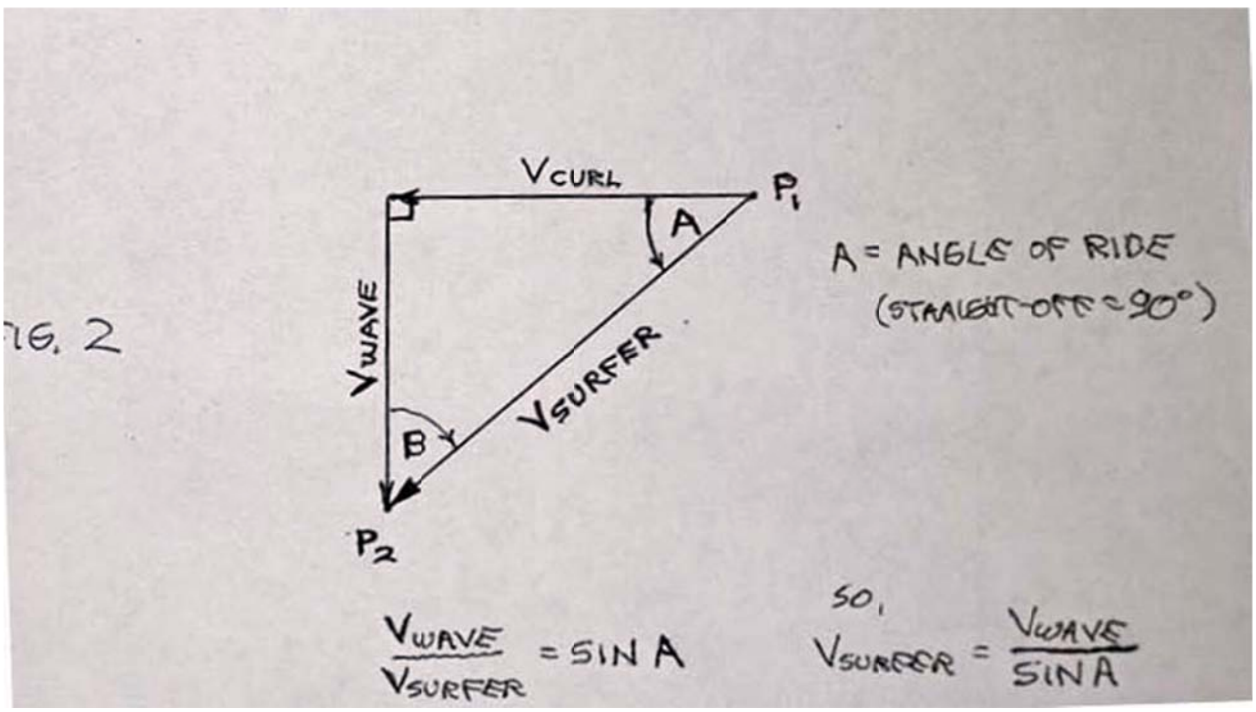
Since it's a 'Right Triangle', Angle C is defined as 90 degrees, and the side opposite C, the hypotenuse, becomes "Side c".

The Pythagorean Theorem says, that for ANY right triangle:

A+B+C= 180 degrees. Also, since C=90 degrees by definition, A+B = 90 degrees.
and,
c squared = a squared + b squared, or, c^2 = a^2 + b^2
and,
the RATIO of the lengths of two sides forming an angle is the same for ALL similar triangles with the same angle between them.
These ratios have been measured and have been named as follows:

Side Opposite/Hypotenuse = the "Sine" of the angle between them
Side Adjacent/Hypotenuse = the "Cosine" of the angle between them
Side Opposite/Side Adjacent = the "Tangent" of the angle between them

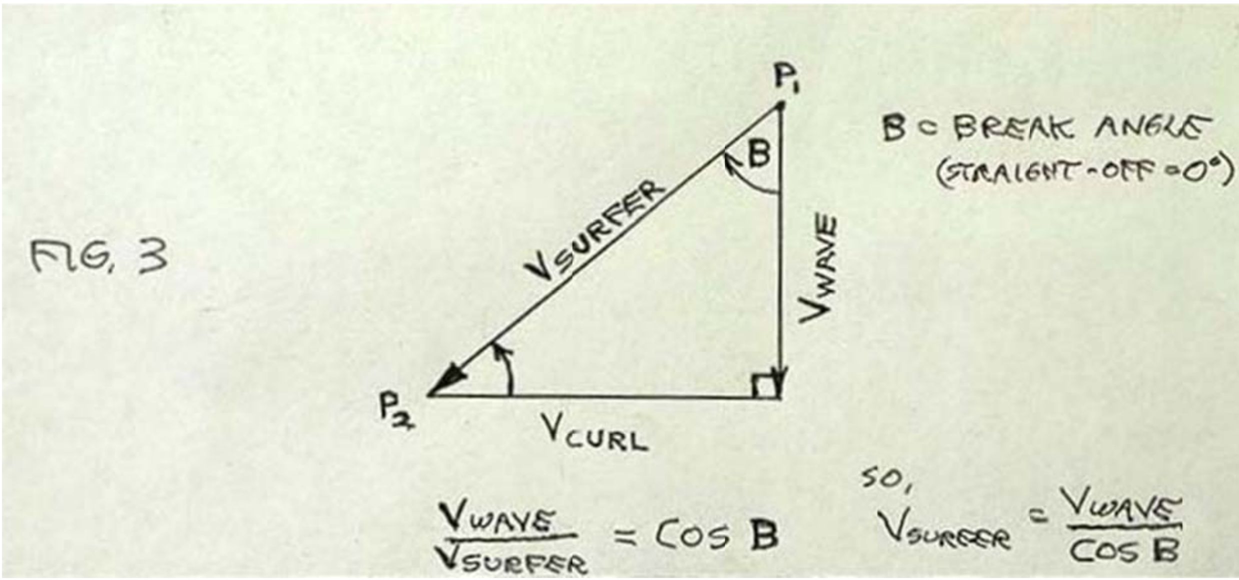
Figure 2. Use if Ride Angle is Measured AWAY from Wave Crest



So, using the right triangle ABC in the example above:

Sin A = a/c and: Sin B = b/c
Cos A = b/c and: Cos B = a/c
Tan A = a/b and: Tan B = b/a

Figure 3. Use if Ride Angle is Measured AWAY from Straight-Off Ride



Now, if we use Angle B for the 'Break Angle', measured in degrees AWAY from going 'straight off', then the following relationship exists between the hypotenuse (Surfer Speed) and the short side (Wave Speed) of our triangle:

Side Adjacent to B/Hypotenuse = Short Side/Hypotenuse = Wave Speed/Surfer Speed = Cos B

solving for "Surfer Speed", we get the following formula:

Surfer Speed = Wave Speed / Cos B. This is my Working Formula!

If the Break Angle = 45 degrees, you would have to go about 1.414 times as fast as the Wave Speed.

If the Break Angle = 60 degrees, you would have to go TWICE as fast as the Wave Speed.

Note that: Cos B = Wave Speed / Surfer Speed
so, Angle B = ArcCosine (Wave Speed / Surfer Speed) or, B=ArcCos(Vwave/Vsurfer)

The "ArcCosine" is the inverse function of the Cosine. It's the ANGLE whose Cosine is known. The possible values of the cosine range from a maximum of 1 for an angle of 0 degrees, down to 0 for an angle of 90 degrees.

The average Break Angle, Ride Angle, or "Peel Angle" B, is "the Angle whose Cosine=(Vwave/Vsurfer). That is, Angle B = ArcCos (Vw/Vs).

So, if you can determine the LENGTH of a surfer's ride, both in Distance and Time, then you could calculate his Average Speed. Then, you would compare his average Ride Speed across the wave with the calculated (or measured) Wave Speed, and from that ratio, you could determine the average Peel Angle or ride angle.

But, how fast is the wave moving in toward the shore while it's in the surf zone? That ONLY depends on how deep the water is under the wave. Bigger waves move faster because they are breaking in deeper water. When the waves get into water that is shallow enough to cause them to break, the wave period or wavelength no longer have much effect on the wave speed. Only the Water Depth in the breaker zone matters.

For the most typical bottom slopes, the water depth "d,ft" where the wave breaks is about 1.28 times the 'TRUE total Wave Height'. The True Wave Height is defined as: the ENTIRE Wave, measured vertically, from top to bottom, 'Crest-to-Trough', so that means INCLUDING the Trough, which may be well out in front of the rideable portion of the wave. You can't use 'Local Scale', 'Slant Height', or whatever, here. Won't work!

Here in Hawaii, the true wave height seems to be about 15-20% greater than the part of the wave that is ABOVE SEA LEVEL, (and therefore can be visually 'measured' by the "Line-of-Sight" method). I use an average of about 1.17 times 'H,asl' to estimate the True Height. That's approximately 7/6, so a wave that measures about 24 ft above sea level is probably more like 28 ft in size, i. e., it has a 4 ft trough (or, 'pit') out in front.

But for 'good' waves, that is, waves with some kind of a tubing shape or pitch-out of the lip, I use the ratio of True Height = 1.2 times the Height above sea level, or "Hasl". If it 'looks like' 20 ft, it's probably closer to 25 ft, including the trough.

For 'Shallow-Water' waves, the Wave Speed, in ft/sec, is given by the formula: V = SQRT(gd), where "g" is the acceleration due to gravitational attraction, in ft/sec^2, and "d" is water depth in feet.

The ratio of "water depth in the breaker zone" / "Breaking Wave Height", or d / Hb, is known as the "Breaker Depth Index", which I will call BDI. Oceanographers like to use a Greek letter for ratios like this. I want to keep it simple. Usually, BDI is given as 1.28, i.e., d = 1.28 times Hb.

Wave Speed at the point of breaking is given as: Vwave = Square Root of (g times d),
or, Vw = SQRT(gd), which can also be written as: Vw = (gd)^0.5

Note that g varies with Latitude, so it is slighter stronger at higher latitudes. That means that waves in colder latitudes are slightly faster than the same-size waves in warmer Tropical waters.

The value of "g" is expressed in units of "meters per second squared" or "feet per second squared".

For readers who don't know why the 'seconds' part of the acceleration formula is squared, it's because the units of motion and distance have to be in the SAME UNITS. Acceleration is the RATE OF CHANGE of speed, which is given here in feet per second. If we used mixed units of measure, we could choose to say that one 'g' is nearly 22 MPH faster 'every second'. If your hotrod could accelerate at that rate with soft slicks instead of street tires, you could get up to 66 MPH in about 3 seconds. That's Superbike territory!

But, when you use the SAME units of measure, say feet and seconds, Velocity is measured in 'feet per second', and you are accelerating at an increasing speed per unit time, or so many 'ft/sec' FASTER every second, so acceleration becomes (ft/sec)/sec, or ft/sec/sec, = ft/(second) squared, = ft/sec^2.

The formula I use for calculating the acceleration of gravity for any given Latitude is as follows:

G = 9.7803267714*((1+0.00193185138639*(sin(LAT))^2)/(SQRT(1-0.00669437999013*(sin(LAT))^2)))

Note that the asterisk (*) stands for "times" (multiplication), and "SQRT" means 'the square root of".

For some calculators, it might be easier to enter this version of the above formula:

G = 9.7803267714*((1+0.00193185138639*(sin(LAT))^2)/((1-0.00669437999013*(sin(LAT))^2)^0.5))

If you have a graphics calculator with a "Solver" function, you can input this formula and solve for either "G" or "LAT", given the value of the other variable. I use a Texas Instruments TI-85 and a TI-92a, but most good graphics calculators have a Solver Function.

If you entered the above equation for "G" correctly, you should get close to the following results:

For 0 degrees Latitude: "G" = 9.7803267714 m/s^2
45 degrees " = 9.806199202464
90 degrees " = 9.832186368574

My own observations over a 15-year period at Makaha (on the "Wild West Side" of Oahu, Hawaii) of wave heights, water depths in the line-ups, and timed length of rides gave me confidence that the above formulas for wave speeds and wave heights vs. breaking depths were pretty accurate. But, I always wondered what the Break Angle was on those big 'Point Break' days (West to WNW Swells, starting at what looks like about 10-12 ft, and rideable up to 25 or 30 ft).

I rode Point Break up to maybe 18 or 20 ft, and when it got bigger, I didn't think I could catch those fast-moving 'Freight Train' swells on my 4'6" paipo board, so I stayed on the beach and took pictures with my 500mm telephoto lens. (Like Dirty Harry said in the movies: "You gotta know your limits!" Ha!).

On smaller days (looks like 12 ft; true height probably around 14-15 ft), you could get a 400 yard ride from the Point line-up to the Bowl that might last a half a minute on a real West swell. That would be about 27 MPH. Piece of cake! A big WNW would be a little faster, but still easily makeable all the way from the Point Line-up to the Bowl.

On BIG days, (looks like 20 ft plus, true height around 25 ft), you need to go 34-35 MPH, and if you have a fast board, you may be able to make it, and your ride might only take about 23 1/2 to 24 seconds. If the wave is almost NW, it will peel off TOO fast to make from the Point, so everybody sits in the next lineup down the line. We called that line-up the "NorthWest Line-up".

But then, on a NW swell, when the wave you're on gets to the Outside Bowl near the end of the ride, the Door gets slammed in your face! HARD!! It can break your board in half if the lip comes down on your board. Don't 'Shoot the Bowl' unless you want to get beat up! West swells let you glide past the Bowl easily. There is a deeper spot just before you get to the Bowl, which we called the "Saddleback". That's where you can get off the wave easily, before it starts to jack up and get out of control. The lip in the Bowl is HEAVY! You don't want to get caught there...

OK, so how fast is a 24-second ride from the Point line-up to the Bowl? It's about 400 yards, or 1200 feet. The average speed would be 1200 ft in 24 seconds, or 50 ft/sec. In Miles Per Hour, the speed is 15/22 times the speed in ft/sec = 34.090909...MPH.

Those were typical ride times that I timed for guys who could make the wave, guys like Buzzy Trent, riding his 11' 0" "Elephant Gun". He could make waves that other guys couldn't.

Those waves were about 25 ft, so what was the Wave Speed? I had calculated the acceleration of gravity at the latitude of Makaha Point lineup as about 32.11 ft/sec squared, so Vwave = (gd)^.5

If d = 1.28 x 25 ft = 32 feet (breaker water depth), then Vw = (g times 32)^.5 = (32.11 x 32)^.5, so Vw = 32.05495282 fps (21.85564965 MPH).

Now, I know that: Cos B = Vwave / Vsurfer = 32.05495282 fps / 50 fps = 0.641099056
So, the wave peels off at a Break Angle B = ArcCosine (0.641099056) = 50.12617774 degrees.
And, that means that the surfer has to go (1/Cos B) times as fast as the wave speed, = 1.559821357 times as fast as the wave. THAT's why big Makaha Point Break is known as a 'Freight Train' wave.

Note: I'm giving the full 10-digit results of the calculations for the guys that want to compare THEIR results with mine to see if their calcs are matching mine. The final result would be rounded off to 2, maybe 3 significant figures. We don't know wave heights to more than 2 significant figures, so we can't know the exact water depth or wave speed, either.

This concludes Part 1 of "Surfer Speed Vs. Wave Speed and Peel (Break) Angle."

Part 2 is an attempt to determine, “HOW FAST can a surfer go on a wave? What's the Maximum Makeable PEEL ANGLE?”

Larry Goddard

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